NOTATION

r_*, z_*, cylindrical coordinates; v_*, w_*, radial and transverse velocity components; $2h_*$, distance between disks; j_*, j_{1*}, rates of sublimation of lower ($z=-1$) and upper ($z=1$) disks; P_*, pressure; ρ_*, density; μ_*, dynamic viscosity; w_{ik}($i=1, 2$), terms of outer asymptotic expansions (9); c_{ik}($i=1, 2$), coefficients of (11); W_k, terms of inner asymptotic expansion (8); z^0, dimensionless coordinate of inflow plane; z_k^0, terms of expansion (10); $R=-j_*h_*/\mu_*$; $R_1=-j_{1*}h_*/\mu_*$; $\chi=j_{1*}/j_*$; $v=v_*\rho_*/j_*$; $w=w_*\rho_*/j_*$; $r=r_*/h_*$; $z=z_*/h_*$; $P=P_*\rho_*/j_*^2$.

LITERATURE CITED

1. A. S. Berman, "Laminar flow in channels with porous walls," J. Appl. Phys., 24, No. 9, 1232-1235 (1953).
2. R. M. Terrill, "Laminar flow in a uniformly porous channel," Aeronaut. Q., 15, 299-310 (1964).
3. R. M. Terrill, "Laminar flow in a uniformly porous channel with large injection," Aeronaut. Q., 16, 323-332 (1965).
4. V. V. Samsonov and D. A. Labuntsov, "Analytical and experimental investigation of hydrodynamics of a vapor flow in a sublimation heat exchanger," Tr. Mosk. Énerg. Inst. Teplo. Massoob. Prots. Appar., No. 198 (1974).
5. A. B. Vasil'eva and V. F. Butuzov, Asymptotic Expansions of Solutions of Singularly Perturbed Equations [in Russian], Nauka, Moscow (1973).
6. M. Van Dyke, Perturbation Methods in Fluid Mechanics, Academic Press, New York (1964).

RHEOLOGICAL PROPERTIES OF HOMOGENEOUS FINELY DISPERSED SUSPENSIONS. STEADY FLOWS

Yu. A. Buevich and I. N. Shchelchkova

UDC 532.529.5:532.135

Expressions are obtained for the rheological parameters (effective viscosity, force of interphase interaction, etc.) of a moderately concentrated suspension of spherical particles. The equations of motion of the suspension and of its phases are written.

A situation when the characteristic spatial scale of the average motion of a dispersed medium is far greater than its internal structural scale, so that it is natural to use the methods of the mechanics of continuous media to describe such motion, is very common in applications. Two fundamental problems arise in this case: to obtain the conservation equations describing the average flow of the phases of the medium and to formulate the rheological equations closing them. In connection with the wide prevalence of dispersed media in various fields of practical activity, both these problems have been discussed in a very large number of reports using the most varied theoretical and experimental methods for media of the most varied types.

For systems consisting of a continuous phase and discrete elements of a dispersed phase distributed in it, the first problem was formally solved in [1, 2] using the well-developed method of averaging of the local conservation equations, which are valid within the materials of the phases, over the ensemble of possible configurations of particles of the dispersed phase. (Bibliographies of research in this field are also presented in the cited reports.) The basic method of solving the second main problem was also indicated in [1, 2], but it was studied concretely only for steady streams of a monodispersed medium containing fine spherical particles in the case when their volume concentration is not too high, so that in averaging over the ensemble one can neglect the nonoverlapping of the solid spheres in the first approximation. In this case the suspension was analyzed as a macroscopic homogeneous "one-velocity" medium.* Analogous problems for the process of heat or mass exchange in a granular medium were considered in [3].

*The results of [1, 2] are presented in Russian in the following preprints: Yu. A. Buevich and I. N. Shchelchkova, "Continuous mechanics of monodispersed suspensions. Conservation equations," and Yu. A. Buevich, B. S. Endler, and I. N. Shchelchkova, "Continuous mechanics of monodispersed suspensions. Rheological equations of state," Preprint Nos. 72 and 85, Inst. Prikl. Mekh. Akad. Nauk SSSR, Moscow (1976) and (1977).

Institute for Problems of Mechanics, Academy of Sciences of the USSR, Moscow.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 5, pp. 872-879, November, 1977. Original article submitted July 1, 1977.

0022-0841/77/3305-1334$07.50

Below we also study a moderately concentrated suspension, which we assume to be macroscopically homogeneous in the sense that the spatial scale of the quantities determining the volume concentration and the granulometric composition of the dispersed phase is considerably larger than the scale of the average velocities of the phases and of the other dynamic variables. However, we use a two-velocity approximation: The difference between the velocities of the phases is constructed, and not only the equations of motion of the suspension as a whole but also of its phases separately are studied in explicit form. The particles are assumed to be spherical, with the Reynolds number for any sphere being assumed to be small. The latter allows one not only to neglect inertial effects in an analysis of the relative motion of the phases but also to ignore the terms due to random pulsations of the phases which appear in the equations of motion in [1, 2] and the possibility of transformation of the kinetic energy of particle rotation into the energy of translational motion. In this case these equations are separated as follows: in a study of the average motion of a suspension there is no need to consider the equations of conservation of the average moment of impulse of its phases. External mass forces with a potential Φ act on the system, but external pairs of forces are assumed to be absent.

First let us consider a monodispersed suspension. Under the adopted assumptions the steady equations of conservation of momentum and mass of the suspension as a whole and of its dispersed phase are written in the laboratory coordinate system $\mathbf{r}$ in the following form [1, 2]:

$$\begin{gathered} d_0\varepsilon(\mathbf{c}_0\nabla)\mathbf{c}_0 + d_1\rho(\mathbf{c}_1\nabla)\mathbf{c}_1 = \nabla\sigma - d\nabla\Phi, \ \nabla\mathbf{c} = 0, \\ d_1\rho(\mathbf{c}_1\nabla)\mathbf{c}_1 = \mathbf{f} - d_1\rho\nabla\Phi, \ \nabla\mathbf{c}_1 = 0. \end{gathered} \tag{1}$$

Here we introduce the average density and velocity of the suspension

$$d = \varepsilon d_0 + \rho d_1, \ \mathbf{c} = \varepsilon\mathbf{c}_0 + \rho\mathbf{c}_1, \tag{2}$$

while σ and $\mathbf{f}$ represent the effective tensor of the average stresses and the average force of the interphase interaction per unit volume of the suspension, which are formally expressed in the form

$$\begin{gathered} \sigma = -\varepsilon p\mathbf{I} + 2\mu_0\mathbf{e} + \rho\sigma_1, \\ \sigma_1 = \frac{3}{4\pi a^3}\int\limits_{x=a} \mathbf{x} * (\mathbf{n}\Sigma)\,d\mathbf{x}, \ \mathbf{f} = n\int\limits_{x=a}(\mathbf{n}\Sigma)\,d\mathbf{x}, \end{gathered} \tag{3}$$

where Σ is the tensor of the average stresses acting on the surface $x=a$ of a single (test) particle of the suspension (the origin of the convective coordinate system $\mathbf{x}$ is taken at the center of this particle) while $\mathbf{e}$ is the tensor of the average deformation velocities in the suspension stream. The equations of conservation of mass and momentum of the continuous phase consist of the differences of the corresponding equations in (1) for the suspension as a whole and for its dispersed phase.

For the determination of the tensor Σ there is the special problem of the flow over the test particle of a two-phase dispersed medium whose properties at a distance from the particle surface coincide with the properties of the suspension but are not homogeneous in the layer adjacent to this surface. For moderately concentrated suspensions the existence of this layer can be neglected entirely, which corresponds to neglecting the effect of nonoverlapping of the particles in the determination of the ensemble of their configurations [1, 2]. In order to formulate this problem, we write the equations of motion (1) in the convective coordinate system

$$\begin{gathered} \nabla\mathbf{v} = 0, \ \nabla\sigma - d\nabla(\Phi + \Psi) = 0, \ \Psi = \mathbf{x}\{(\mathbf{c}_1\nabla)\mathbf{c}_1\}_{x=0}, \\ \nabla\mathbf{v}_1 = 0, \ \mathbf{f} - d_1\rho\nabla(\Phi + \Psi) = 0, \end{gathered} \tag{4}$$

where the term containing $(\mathbf{c}_1\nabla)\mathbf{c}_1$ reflects the appearance of an inertial force in this coordinate system and is calculated for the point $x=0$ connected with the center of the test particle. In writing (4) we neglected the inertial effects connected with the relative motion of the phases.

In view of the linearity of Eqs. (4), one can take [1, 2]

$$\sigma = -p\mathbf{I} + 2\mu\mathbf{e}_0, \ \mathbf{f} = k_1(\mathbf{v}_0 - \mathbf{v}_1) + k_2\Delta\mathbf{v}_0 + k_3\nabla(\Phi + \Psi). \tag{5}$$

Here μ and k_i ($i=1, 2, 3$) are some unknown coefficients which are determined a posteriori from the condition of self-consistency of the theory, namely, from the condition of agreement of Eqs. (5) with the equations which follow from (3) after the solution of the test-particle problem and the calculation of the integrals in (3) on the basis of this solution. We emphasize that here the shear stresses in the suspension are considered in the approximation, usual for the mechanics of homogeneous fluid media, when one only allows for the contributions to these stresses proportional to the first powers of the first-order derivatives of the velocity components with respect to the coordinates [4]. Therefore, the condition of agreement of the stresses from (3) and (5) is understood to be only with the accuracy of terms of just such a type, while the condition of agreement of the expressions for the force is with the accuracy of terms linear with respect to the second-order derivatives [actually, it is clear from Eqs. (4) that the orders of magnitude of the components of the vectors **f** and $\nabla\sigma$ are the same].

From (4) and (5) and the boundary conditions of attachment at the surface of the test particle we have the following problem on the flow over it:

$$\nabla \mathbf{v}_0^* = 0, \quad -\nabla p^* + \mu\Delta\mathbf{v}_0^* - d\nabla(\Phi + \Psi) = 0, \quad \mathbf{v}_0^* = \boldsymbol{\omega}\times\mathbf{x}, \quad x = a;$$
$$\mathbf{v}_0^*, \; p^* \to \mathbf{v}_0, \; p, \; x \to \infty. \tag{6}$$

At an infinite distance from the particle the perturbed velocity $\mathbf{v}_0^*$ and the perturbed pressure p* of the continuous phase must change into the corresponding unperturbed quantities $\mathbf{v}_0$ and p (here "infinity" is understood in the sense of the method of joined asymptotic expansions). The solution of the problem (6) is easily expressed, with arbitrary $\mathbf{v}_0$ and p satisfying Eqs. (4), in the form of series with respect to basis functions built on spherical harmonics (see [5, 6], for example); here we use the solution in the form obtained in [7], where some important integrals over the surface of the test particle were also calculated.

Using the results of [7] and defining Σ in accordance with the equation for σ in (5), for the quantity in (3) we have

$$\rho\sigma_1 = \rho(-p\mathbf{I} + 5\mu\mathbf{e}_0 + 1/2\,\mu a^2\Delta\mathbf{e}_0), \tag{7}$$

where the last term in the definition of σ should be discarded so as not to assume an excess accuracy (see the comment above). Similarly, after integration [7] we have from (3)

$$\mathbf{f} = \frac{9}{2}\frac{\rho\mu}{a^2}(\mathbf{v}_0 - \mathbf{v}_1) + \frac{3}{4}\rho\mu\Delta\mathbf{v}_0 + \rho d\nabla(\Phi + \Psi), \tag{8}$$

which leads [after comparison with (5)] to the equalities

$$k_1 = \frac{9}{2}\frac{\rho\mu}{a^2}, \quad k_2 = \frac{3}{4}\rho\mu, \quad k_3 = \rho d. \tag{9}$$

Further, from the equation of conservation of momentum of the dispersed phase in (4), with allowance for the expressions for the force in (8) and (9), we have

$$\mathbf{v}_1 = \mathbf{v}_0 + \frac{1}{6}a^2\Delta\mathbf{v}_0 + \frac{2}{9}\frac{a^2(d - d_1)}{\mu}\nabla(\Phi + \Psi), \tag{10}$$

from which, with the accuracy of higher derivatives of the velocity, we have

$$\mathbf{e} = \varepsilon\mathbf{e}_0 + \rho\mathbf{e}_1 = \mathbf{e}_0 = \mathbf{e}_1, \quad \Delta\mathbf{v} = \Delta\mathbf{v}_0 = \Delta\mathbf{v}_1 \tag{11}$$

[the order of magnitude of the terms arising in the differentiation of (10) is easily estimated using Eqs. (4)].

Allowing for (7) and (11), and comparing the expressions for σ in (3) and (5), we obtain the equation for the effective viscosity of the suspension

$$\mu = \mu_0\left(1 - \frac{5}{2}\rho\right)^{-1}. \tag{12}$$

As would be expected, this quantity coincides with that determined on the basis of a one-velocity model in [1, 2], as well as in [8]. Equation (12) was obtained earlier in [9] by another method.

Equations (5), (9), and (12) close the system of equations (1) or (4) for macroscopically homogeneous suspensions of identical fine spheres. In accordance with (11) the tensor of the deformation velocities of the suspension can be identified with those for the continuous or dispersed phases [so that $\mathbf{e}_0$ and $\Delta\mathbf{v}_0$ in (5) can be replaced by **e** and $\Delta\mathbf{v}$], the relative velocity $\mathbf{v}_0 - \mathbf{v}_1$ in the convective coordinate system obviously coincides with the velocity $\mathbf{c}_0 - \mathbf{c}_1$ in the laboratory system, while the spatial derivatives of the velocities are the same in the two coordinate systems.

A comparison of Eq. (12) with the experimental data of [10-12] is illustrated in Fig. 1. The tests of [10, 11] were chosen for comparison because they are usually used to test the majority of proposed equations, empirical and others, while the tests of [12] were chosen in view of the very pure conditions under which they were conducted. Only the experimental results of [12] for suspensions of polymethyl methacrylate particles in aqueous solutions of glycerin, obtained on a rotary viscosimeter, are shown in Fig. 1. Both the stability of the suspension (particle aggregates were not formed) and the absence of structure formation (characteristic of flows in a capillary viscosimeter, for example) were guaranteed in these experiments. It is seen that the theory, developed for moderately concentrated suspensions, leads to fully satisfactory results up to a volume concentration of particles equal to 20-25%. Curves corresponding to the well-known Einstein equation and the equation of [13] are also presented in Fig. 1.

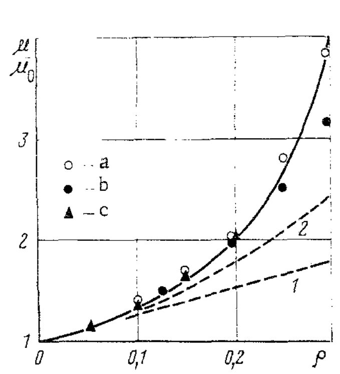

Fig. 1

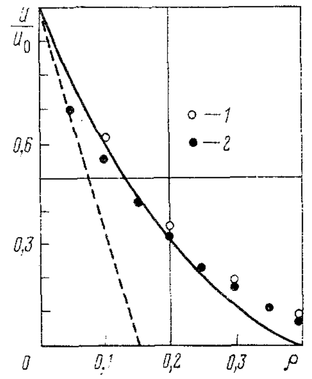

Fig. 2

Fig. 1. Dependence of relative viscosity of a suspension on its volume concentration; solid curve) Eq. (12); dashed curves) equations of Einstein (1) and of Batchelor and Green [13] (2); a) data of [10]; b) of [11]; c) of [12].

Fig. 2. Dependence of relative velocity of sedimentation on volume concentration of suspension: solid curve) Eq. (14); dashed curve) Batchelor's equation [16]; 1) experimental data of [14]; 2) of [15].

Now let us consider the effects connected with the relative motion of the phases, and consequently with their interaction. The theoretical description of such effects will obviously depend essentially on whether one uses the simple one-velocity model for this (as in [1, 2, 8], where the flow over the test particle of a homogeneous medium modeling the suspension as a whole was analyzed) or the realistic two-velocity model (as above).

First of all, the force of the interphase interaction in (8) differs from that calculated in [1, 2, 8] in the respect that the average velocity $\mathbf{v}_0$ (or $\mathbf{c}_0$) of the continuous phase figures in (8) in place of the velocity $\mathbf{v}$ (or $\mathbf{c}$) of the suspension. This is entirely unimportant in a description of the motion of the suspension as a whole, but very important in an analysis of phenomena such as the settling of suspensions in channels, holding tanks, etc. In fact, let us calculate the particle sedimentation rate $\mathbf{u}=\mathbf{c}_1$, for example, in a gravitational field ($\nabla\Phi=-\mathbf{g}$) with the condition that the suspension as a whole is stationary, i.e., $\mathbf{c}=0$. In this case, from (2) we have $\mathbf{c}_0=-(\rho/\varepsilon)\mathbf{u}$, and the equation of conservation of momentum of the dispersed phase from (1) gives

$$\frac{9}{2}\,\frac{\rho\mu}{a^2}\,\frac{\mathbf{u}}{\varepsilon}=\rho\,(d_1-d)\,\mathbf{g}. \tag{13}$$

From this and from the definitions of d in (2) and μ in (12) we obtain

$$\mathbf{u}=(1-\rho)^2\left(1-\frac{5}{2}\,\rho\right)\mathbf{u}_0,\quad \mathbf{u}_0=\frac{2}{9}\,\frac{a^2(d_1-d_0)\,\mathbf{g}}{\mu_0}, \tag{14}$$

where u_0 is the Stokes velocity of settling of a particle in the purely continuous phase, the density and viscosity of which are d_0 and μ_0, respectively.

In Fig. 2 Eq. (14) is compared with the experiments in [14, 15]. The curve which follows from [16] is also shown here. It is seen that the agreement between theory and experiment is good enough up to values of ρ equal to 0.20-0.25.

Let us indicate the meaning of the individual terms in (8). The first term describes the force of the viscous action of the surrounding medium on the particle per unit volume of suspension; in this case the effective viscosity μ of the suspension plays the role of the viscosity of the medium, while the true velocity of interphase slippage $\mathbf{v}_0-\mathbf{v}_1$ (or $\mathbf{c}_0-\mathbf{c}_1$) plays the role of the relative velocity. The second term represents the effective Faxén force, also determined with the help of the viscosity μ. Finally, the third term in (8) describes the effective buoyant force (the Archimedes force), due both to the external mass force field and to the inertial force field; in this case the average density d of the suspension figures as the density of the external medium.

We note that the discussion of how to correctly write the individual terms in the expression for the force of the interphase interaction, particularly those connected with the pressure gradient in an unperturbed stream, continues at present (see [17], for example), with a distinction being made between the components of this gradient produced by hydrostatic and hydrodynamic causes. Equation (8) makes it possible to finish this discussion in application to steady streams containing fine particles.

We can show that there are a number of problems in which the correct allowance for the components of the interaction force proves important not only in a quantitative but even in a qualitative respect. For example, it is shown in [18] that the introduction of a term of the form $-\rho\nabla p$ into this force seriously alters the pattern of inertial settling of particles of a suspension onto a body over which it flows in comparison with the pattern predicted in the case when such a term is ignored. In fact, a term proportional to $\Delta \mathbf{v}_0$ figures in (8) in place of the term containing ∇p, and while the former quantity can be expressed through other dynamic variables using the equations of motion, its final expression still differs from that postulated in [18].

Up to now we have considered the equations of conservation of mass and momentum. In a number of cases it is also important to study the effect of the components of the effective stress tensor, which depend on the angular velocities of rotation of the particles, as well as the equations of conservation of the moment of impulse of the phases. Formal representations for these component stresses and equations are presented in [1, 2]. The latter include two quantities which are also expressed through integrals over the surface of the test particle of the stresses acting on it: the average moment of the interphase interaction and the pseudotensor of the average "moment" stresses. From the solution of the problem (6), after calculations, we obtain

$$\mathbf{m} = n \int\limits_{x=a} \mathbf{x} \times (\mathbf{n}\Sigma)\, d\mathbf{x} = 6\rho\mu \left(\frac{1}{2} \operatorname{rot} \mathbf{v}_0 - \boldsymbol{\omega}\right), \tag{15}$$

as well as

$$\boldsymbol{\varkappa} = n \int\limits_{x=a} \mathbf{x} * [\mathbf{x} \times (\mathbf{n}\Sigma)]\, d\mathbf{x} + \frac{1}{5} a^2 \boldsymbol{\varepsilon}\mathbf{f} = 2\rho a^2 \mu \mathbf{y} + \frac{1}{5} a^2 \boldsymbol{\varepsilon}\mathbf{f}. \tag{16}$$

Here $\boldsymbol{\varepsilon}$ is the antisymmetric alternating Levi-Civita tensor while $\mathbf{y}$ is the pseudotensor of the "deformation velocities," constructed from the field of the pseudovector $\operatorname{rot} \mathbf{v}_0$. It is obvious that with the adopted accuracy $\operatorname{rot} \mathbf{v}_0 = \operatorname{rot} \mathbf{v} = \operatorname{rot} \mathbf{c}_0 = \operatorname{rot} \mathbf{c}$.

Monodispersed suspensions have been discussed above. Now let us generalize the results obtained to a polydispersed suspension with a size distribution function $\varphi(a)$ of the particles such that

$$n = \int \varphi(a)\, da, \quad \rho = \frac{4}{3} \pi \int a^3 \varphi(a)\, da. \tag{17}$$

It is easy to see that such a generalization is trivial for the moderately concentrated suspensions under consideration. In fact, the polydispersion of the particles affects the rheological characteristics of the medium only to the extent to which it affects the properties of the fictitious dispersed medium flowing over the test particle in the surface layer concentric with the particle [1, 2]. But in the neglect of the nonoverlapping of the particles the existence of such a layer was neglected altogether. Therefore, the results obtained above will also be valid for a particle of any radius in a polydispersed suspension. It is clear, in particular, that in the approximation under consideration the effective viscosity of the suspension does not depend at all on the form of the function $\varphi(a)$ and is determined by Eq. (12) as before. The force acting on the $\varphi(a)da$ particles which are in a unit volume of the suspension and have radii in the interval $(a, a+da)$ on the part of the surrounding medium has the form (we use the laboratory coordinate system)

$$d\mathbf{f} = \mathbf{F}(a)\, \varphi(a)\, da \left[6\pi a\mu (\mathbf{c}_0 - \mathbf{c}_1(a)) + \pi a^3 \mu \Delta \mathbf{c}_0 + \frac{4}{3} \pi a^3 d \nabla (\Phi + \Psi(a))\right] \varphi(a)\, da. \tag{18}$$

The velocity $\mathbf{c}_1(a)$ of the particles of radius a is determined from the equations of conservation of mass and momentum for such particles, which are fully analogous to the equations in (1)

$$\nabla \mathbf{c}_1(a) = 0, \quad \frac{4}{3} \pi a^3 d_1 (\mathbf{c}_1(a) \nabla)\, \mathbf{c}_1(a) = \mathbf{F}(a) - \frac{4}{3} \pi a^3 d_1 \nabla \Phi. \tag{19}$$

In this case the equations of conservation of mass and momentum for the suspension as a whole have the form [we use the representation for σ from (5)]

$$\nabla \mathbf{c}_0 = 0, \quad d_0 \varepsilon (\mathbf{c}_0 \nabla)\, \mathbf{c}_0 + \frac{4}{3} \pi d_1 \int a^3 (\mathbf{c}_1(a) \nabla)\, \mathbf{c}_1(a)\, \varphi(a)\, da = -\nabla p + \mu \Delta \mathbf{v}_0 - d \nabla \Phi. \tag{20}$$

The system of equations (19) and (20) serves for the determination of the unknowns $\mathbf{c}_0$, $\mathbf{c}_1(a)$, and p [in a stream of macroscopically homogeneous suspension the equations of conservation of mass in (19) and (20) are a consequence of one another]. The equations of conservation of mass and momentum for the continuous phase are obtained after the subtraction from (20) of the corresponding equations of (19), integrated over da with a weight $\varphi(a)$.

By an entirely analogous means it is easy to obtain expressions for the moment acting on particles of a given radius per unit volume on the part of the surrounding medium and for the effective pseudotensor of the moment stresses figuring in the equations of conservation of the moment of impulse [1, 2], which replace (15) and (16) in the case of a polydispersed suspension.

In conclusion, we note that it is also easy to write the expressions for the various characteristics of a suspension which is polydispersed not only by the size but also by the density of the particles contained in it.

NOTATION

a, particle radius; **c**, velocity in the laboratory coordinate system **r**; d, density; **e**, tensor of deformation velocities; **f**, **F**(a), force of interphase interaction and density of its distribution with respect to a; **g**, acceleration of gravity; k_i, coefficients in (5) and (9); **m**, moment of interphase interaction; n, numerical concentration of particles; **n**, unit vector of outer normal at surface of test particle; p, pressure; **u**, $\mathbf{u}_0$, velocity of sedimentation of the suspension and of a single particle, respectively; **v**, velocity in the convective coordinate system **x** connected with the center of the test particle; **y**, pseudotensor of deformation velocities constructed with respect to rot $\mathbf{v}_0$; $\varepsilon = 1 - \rho$; $\varkappa$, pseudotensor of moment stresses; μ, viscosity; ρ, volume concentration of particles; σ, Σ, stress tensors; Φ, potential of external mass forces; $\varphi(a)$, particle distribution function by radii; Ψ, potential of inertial forces, introduced in (4); $\boldsymbol{\omega}$, angular velocity of particle rotation; the subscripts zero and one pertain to the continuous and dispersed phases, respectively; an asterisk denotes the dyadic multiplication of vectors; an asterisk above marks the fields perturbed by the test particle.

LITERATURE CITED

1. Yu. A. Buevich (Buyevich), Z. Angew. Math. Mekh., 56, T379 (1976).
2. Yu. A. Buevich (Buyevich) and I. N. Schchelchkova, in: Progress in Aerospace Sciences, Vol. 17, Pergamon Press, London (1977).
3. Yu. A. Buevich, Yu. A. Korneev, and I. N. Shchelchkova, Inzh.-Fiz. Zh., 30, 979 (1976).
4. L. D. Landau and E. M. Lifshits, Mechanics of Continuous Media [in Russian], Gostekhteorizdat, Moscow (1954).
5. H. Lamb, Hydrodynamics, 6th ed., Cambridge University Press, London (1932).
6. J. Happel and H. Brenner, Low Reynolds Number Hydrodynamics, 2nd ed., Noordhoff International Publishing Company, Leiden (1973).
7. Yu. A. Buevich and V. G. Markov, Prikl. Mat. Mekh., 36, 480 (1972).
8. T. S. Lundgren, J. Fluid Mech., 51, 273 (1972).
9. A. D. Maude, J. Fluid Mech., 7, 230 (1960).
10. I. R. Rutgers, Rheol. Acta, 2, 202 (1962).
11. D. G. Thomas, J. Colloid Sci., 20, 267 (1965).
12. S. Iwanami and M. Tachibana, Bull. Jpn. Soc. Mech. Eng., 12, 224 (1969).
13. G. K. Batchelor and J. T. Green, J. Fluid Mech., 56, 401 (1972).
14. T. S. Hanratty and A. Bandukwala, Am. Inst. Chem. Eng. J., 3, 293 (1957).
15. T. Shannon, E. Stroupe, and E. M. Tory, Ind. Eng. Chem. Fund., 2, 203 (1963).
16. G. K. Batchelor, J. Fluid Mech., 52, 245 (1972).
17. S. L. Soo, Phys. Fluids, 18, 263 (1975).
18. D. A. Drew, Phys. Fluids, 17, 1688 (1974).